POG Modeling of Automotive Systems

MORE on Automotive - 28 Maggio 2018

Prof. Roberto Zanasi
Graphical Modeling Techniques

**Graphical Techniques** for representing the dynamics of physical systems:

1) Bond-Graph (BG) – (1959)
2) **Power-Oriented Graphs (POG)** - 1991
3) Energetic Macroscopic Representation (EMR) – 2001
4) Causal Ordering Graphs (COG) - 2000
5) Vectorial Bond Graphs (VBG)
6) … (F.d.T., G.f.s., ecc.)

The BG, POG, EMR and VBG techniques put in evidence the power flows that are present within the system.

This presentation shows how the POG Technique can be useful for modeling Automotive Subsystems.
Power-Oriented Graphs (POG)

The Power-Oriented Graphs are "block diagrams" obtained by using a "modular" structure essentially based on the following two blocks:

- **Elaboration block**
  - 3-Port Junctions:
    - 0-Junction;
    - 1-Junction;
  - 1-Port Elements:
    - Capacitor C;
    - Inertia I;
    - Resistor R;

- **Connection block**
  - Positive power flows
  - 2-Port Elements:
    - Transformers TR;
    - Girators GY;
    - Modulated TR;
    - Modulated GY;

- Power sections: the POG maintain a direct correspondence between pairs of variables and real power flows: the product of the two variables involved in each dashed line has the meaning of "power flowing through that section".

- The Elaboration block can store and dissipate/generate energy.

- The Connection block can only "transform" the energy.
POG modeling: a simple example

Across and through variables:

<table>
<thead>
<tr>
<th>Domains</th>
<th>Across: ( v_e )</th>
<th>Through: ( v_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>Voltage ( V )</td>
<td>Current ( I )</td>
</tr>
<tr>
<td>Mec. Tra.</td>
<td>Velocity ( v )</td>
<td>Force ( F )</td>
</tr>
<tr>
<td>Mec. Rot.</td>
<td>Ang. Vel. ( \omega )</td>
<td>Torque ( \tau )</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure ( P )</td>
<td>Flow rate ( Q )</td>
</tr>
</tbody>
</table>

Across variables are defined between two points:

Through variables are defined in a single point:

Kirchhoff's current law

\[
\begin{align*}
\frac{1}{C} \frac{dV_c}{dt} + I_R &= 0 \\
R \cdot I_R &= V_o
\end{align*}
\]

The system

The POG model
POG: DC motor with an hydraulic pump

A DC motor connected to an hydraulic pump:

There is a direct correspondence between the POG blocks and the physical elements …

The POG model:

POG models can be directly inserted in Simulink.
The POG state space description of the DC motor with hydraulic pump:

\[
\begin{bmatrix}
L_a & 0 & 0 \\
0 & J_m & 0 \\
0 & 0 & C_0
\end{bmatrix}
\begin{bmatrix}
\dot{I}_a \\
\dot{\omega}_m \\
\dot{P}_0
\end{bmatrix}
= \begin{bmatrix}
-R_a & -K_m & 0 \\
K_m & -b_m & -K_p \\
0 & 0 & -\alpha_p
\end{bmatrix}
\begin{bmatrix}
I_a \\
\omega_m \\
P_0
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
V_a \\
Q_0
\end{bmatrix}
\]

The coefficients of matrices \( A \) and \( B \) are the gains of all the paths that link the state and input variables \( \mathbf{x} \) and \( \mathbf{u} \) to the inputs of the integrators:

The elements of matrix \( L \) are the coefficients of the constitutive relations.
The POG model of the DC motor with hydraulic pump:

\[
\begin{bmatrix}
L & \dot{x} \\
\dot{L} & \ddot{x}
\end{bmatrix}
\begin{bmatrix}
I_a \\
\dot{I}_a
\end{bmatrix}
= \begin{bmatrix}
-R_a -K_m & 0 \\
K_m & -b_m -K_p
\end{bmatrix}
\begin{bmatrix}
I_a \\
\dot{I}_a
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
V_a \\
Q_0
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
\]

The POG models can be represented by the compact vectorial scheme:

\[
L \dot{x} = -A x + B u \\
y = C x + D u
\]

Stored Energy: \( E_s = \frac{1}{2} x^T L x \)

Dissipating Power: \( P_d = x^T A x \)

\( A_s \) contains only the dissipating elements:

\[
A_s = \frac{A + A^T}{2} = \begin{bmatrix}
R_a & 0 & 0 \\
0 & b_m & 0 \\
0 & 0 & \alpha_p
\end{bmatrix}
\]

\( A_w \) contains only the connection elements:

\[
A_w = \frac{A - A^T}{2} = \begin{bmatrix}
0 & K_m & 0 \\
-K_m & 0 & K_p \\
0 & -K_p & 0
\end{bmatrix}
\]
When an eigenvalue of matrix \( L \) goes to zero (or to infinity), the system degenerates towards a lower dynamic dimension system. The “reduced system” can be obtained by using a congruent transformation \( x = Tz \) where \( T \) is a rectangular matrix:

\[
\begin{align*}
\begin{cases}
T^TLTz &= T^TATz + T^TBu \\
y &= B^Tz
\end{cases}
\end{align*}
\Rightarrow \begin{cases}
\overline{L}z &= \overline{A}z + \overline{B}u \\
y &= \overline{B}^Tz
\end{cases}
\]

When a parameter goes to zero a static relation between state variable arises:

\[
J_m = 0 \quad \Rightarrow \quad K_m I_a - b_m \omega_m - K_p P_0 = 0 \quad \Rightarrow \quad \omega_m = \frac{K_m}{b_m} I_a - \frac{K_p}{b_m} P_0
\]

A rectangular state space transformation can be easily obtained:

\[
\begin{bmatrix}
I_a \\
\omega_m \\
P_0
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
\frac{K_m}{b_m} & -\frac{K_p}{b_m} & 0 \\
0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
I_a \\
P_0
\end{bmatrix}
\]

Applying the congruent transformation one directly obtains the reduced system:

\[
\begin{bmatrix}
L_a & 0 \\
0 & \frac{1}{K_0}
\end{bmatrix} \begin{bmatrix}
\dot{I}_a \\
\dot{P}_0
\end{bmatrix} = \begin{bmatrix}
-R_a - \frac{K_m^2}{b_m} & \frac{K_m K_p}{b_m} \\
\frac{K_m K_p}{b_m} & -\alpha p - \frac{K_p^2}{b_m}
\end{bmatrix} \begin{bmatrix}
I_a \\
P_0
\end{bmatrix} + \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} \begin{bmatrix}
V_a \\
Q_0
\end{bmatrix}
\]
Modeling a Planetary Gear

The POG blocks have a direct correspondence with the physical elements.

The POG model:
Planetary Gear: State Space Model

From the POG block scheme directly follows the dynamic state space model of the system (n=6):

\[
\dot{\bar{L}} \dot{\bar{x}} = -\bar{A} \bar{x} + \bar{B} u, \\
y = \bar{B}^T \bar{x}
\]

**State vector:**
\[
\bar{x} = \begin{bmatrix}
\omega_s \\
F_{sc} \\
\omega_p \\
\omega_c \\
F_{cr} \\
\omega_r
\end{bmatrix}
\]

**Energy matrix:**
\[
\bar{L} = \begin{bmatrix}
J_s & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{K_{sc}} & 0 & 0 & 0 & 0 \\
0 & 0 & J_p & 0 & 0 & 0 \\
0 & 0 & 0 & J_c & 0 & 0 \\
0 & 0 & 0 & \frac{1}{K_{cr}} & 0 & 0 \\
0 & 0 & 0 & 0 & J_r & 0
\end{bmatrix}
\]

**Input matrix:**
\[
\bar{B} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Power matrix:
\[
\bar{A} = \begin{bmatrix}
-b_s - r_s^2 d_{sc} & -r_s & -r_s d_{sc} r_p & r_s^2 d_{sc} & 0 & 0 \\
r_s & 0 & 0 & 0 & 0 & 0 \\
-r_s d_{sc} r_p & -r_p & -b_p - d_{sc} r_p^2 - d_{cr} r_r^2 & r_s d_{sc} r_p - d_{cr} r_p r_r & -r_p & d_{cr} r_p r_r \\
r_s^2 d_{sc} & r_s & r_s d_{sc} r_p - d_{cr} r_p r_r & -b_c - r_s^2 d_{sc} - d_{cr} r_r^2 & -r_r & d_{cr} r_r^2 \\
0 & 0 & r_p & d_{cr} r_p r_r & 0 & -r_r \\
0 & 0 & d_{cr} r_p r_r & 0 & r_r & -b_r - d_{cr} r_r^2
\end{bmatrix}
\]

Is this model too complex? POG technique provides the reduced models when the stiffness tend to infinity or the inertias tend to zero.
**Planetary Gear: Reduced Model**

When the stiffness coefficients tend to infinity, a static relation between the state variables appears:

\[
\begin{bmatrix}
\omega_p \\
\omega_r
\end{bmatrix} = -A_{32}^{-1} A_{31} \begin{bmatrix}
x_1
\end{bmatrix}
\]

Applying a **rectangular and congruent state space transformation**

\[
T_2 = \begin{bmatrix}
I_2 \\
-A_{32}^{-1} A_{31}
\end{bmatrix}
\]

one obtains the transformed and reduced system:

\[
L_r \dot{x}_1 = -A_r x_1 + B_r u \\
y = B_r^T x_1
\]

Matrices \( L_r = T_2^T L T_2 \), \( A_r = T_2^T A T_2 \) and \( B_r = T_2^T B \) have the following structure:

\[
L_r = \begin{bmatrix}
J_s + \frac{r_s^2}{r_p^2} J_p + \frac{r_s^2}{r_r^2} J_r \\
-\frac{r_s^2}{r_r^2} J_p - \frac{r_s}{r_r} \left( 1 + \frac{r_s}{r_r} \right) J_r \\
-\frac{r_s^2}{r_p^2} J_p - \left( 1 + \frac{r_s}{r_p} \right) J_r
\end{bmatrix}
\]

\[
A_r = \begin{bmatrix}
b_s + \frac{r_s^2}{r_p^2} b_p + \frac{r_s^2}{r_r^2} b_r \\
-\frac{r_s^2}{r_p^2} b_p - \frac{r_s}{r_r} \left( 1 + \frac{r_s}{r_r} \right) b_r \\
-\frac{r_s^2}{r_p^2} b_p - \left( 1 + \frac{r_s}{r_p} \right) b_r
\end{bmatrix}
\]

\[
B_r = \begin{bmatrix}
1 & 0 & -\frac{r_s}{r_p} \\
0 & 1 & 1 + \frac{r_s}{r_r}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\omega_s \\
\omega_c \\
\omega_r
\end{bmatrix} = \begin{bmatrix}
\omega_s \\
\omega_r
\end{bmatrix}
\]
Planetary Gear Reduced Model: Simulation Results

Comparison between the extended and reduced models:

Parameters used in simulation (SI):

\[
\begin{align*}
[J_s, b_s, r_s] &= [0.049, 4.946, 0.102] \\
[J_r, b_r, r_r] &= [2.180, 218.02, 0.248] \\
[J_p, b_p, r_p] &= [0.081, 8.123, 0.073] \\
[J_c, b_c] &= [0.929, 92.89] \\
[K_{sc}, d_{sc}] &= [K_{cr}, d_{cr}] = [10^7, 10] \\
[\tau_s, \tau_c, \tau_r] &= [0, 4, 0] \\
[\omega_s(0), \omega_c(0), \omega_r(0)] &= [0.1, -0.2, -0.323]
\end{align*}
\]

Note: the use of a “rectangular” matrix for transforming and reducing a dynamical system is a “specific characteristics” of the POG technique.
Continuous Variable Transmission (CVT)

Equivalent inertia (CVT reduced system):

\[ J_t(\theta) = J_0 + J_m + \frac{J_p r_m^2}{r_p^2} + \frac{(h_p(\theta))^2 J_q r_q^2}{h_q r_p^2} + \frac{(h_p(\theta))^2 J_r r_r^2}{h_q r_p^2} + \frac{(h_p(\theta))^2 J_e r_e^2}{h_q r_p^2} + \]

\[ + \frac{J_s(\theta)(h_p(\theta)r_p r_m r_q r_r + h_q r_p r_e r_e)^2}{(h_q r_p r_q r_r + h_q r_p r_e r_e)^2} \]

Equivalent friction coefficient (CVT reduced system):

\[ b_t(\theta) = \frac{1}{h_q r_p^2} (b_i (t_p(\theta))^2 r_m^2 r_q^2 + b_i h_p(\theta)^2 r_m r_q + b_i h_p(\theta)^2 r_m r_e + b_i h_p(\theta)^2 r_m r_e) + \]

\[ + \frac{1}{(r_m + r_e)^2} (r_0 r_m (b_0 r_0 - b_e r_e) ((h_p(\theta))^2 r_m r_q r_e + \]

\[ + b_0 h_p(\theta) r_m r_q r_e + b_e (h_p(\theta))^2 r_m r_e)^2 + h_q r_p r_e (-h_q r_p r_e + 2 h_p(\theta) r_m r_q r_e))) \]
KERS - Full Toroidal Variator (FTV)

Extended POG model with fictitious stiffnesses:

\[ \tau_1 \rightarrow r_1 \rightarrow \frac{1}{s} b_1 \rightarrow J_1^{-1} \rightarrow r_1 \rightarrow \frac{1}{s} K_{12} \rightarrow \frac{1}{s} J_2^{-1} \rightarrow r_2 \rightarrow \frac{1}{s} b_2 \rightarrow J_2^{-1} \rightarrow r_2 \rightarrow \frac{1}{s} K_{2b} \rightarrow \frac{1}{s} J_3^{-1} \rightarrow r_3 \rightarrow \frac{1}{s} b_3 \rightarrow J_3^{-1} \rightarrow r_3 \rightarrow \tau_3 \]
When the stiffnesses tend to infinity, \(K_{12} \to \infty, K_{2b} \to \infty\) and \(K_{b3} \to \infty\), the POG model reduces to a one-dimensional time-varying dynamic system:

**Congruent Transformation:**

\[
x := \begin{bmatrix}
1 & 0 \\
0 & R_2 \\
R_b(\theta) & 0 \\
R_3(\theta)
\end{bmatrix} \omega_1
\]

where:

\[R_2 = \frac{r_1}{r_2},\]
\[R_b(\theta) = \frac{r_1 (r_a + r_b \sin \theta)}{r_2 r_b},\]
\[R_3(\theta) = \frac{r_1 (r_a + r_b \sin \theta)}{r_2 (r_a - r_b \sin \theta)}.\]

**Full Toroidal Variator: Reduced Model**

\[J(\theta) = J_1 + R_2^2 J_2 + R_b^2(\theta) J_b + R_3^2(\theta) J_3\]
\[b(\theta) = b_1 + R_2^2 b_2 + R_b^2(\theta) b_b + R_3^2(\theta) b_3\]
\[N(\theta, \dot{\theta}) = r_{2\theta} r_b \dot{\theta} \cos \theta R_2^2 \left( \frac{J_b}{r_b} + \frac{2 r_a J_3}{r_3^2} \right)\]
POG model of Multi-Phase Synchronous Motors

POG state space model:

\[
\begin{bmatrix}
\omega L & 0 \\
0 & J_r
\end{bmatrix}
\begin{bmatrix}
\omega \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{\omega} \\
\dot{\theta}
\end{bmatrix}
= -\begin{bmatrix}
\omega R + J_\omega \omega L & \omega K_T \\
-\omega K^T_T & b_r
\end{bmatrix}
\begin{bmatrix}
\dot{\omega} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
\omega V \\
-\tau_e
\end{bmatrix}
\]

POG block scheme:

The POG scheme is very compact and can be easily translated into a Simulink model.

\( \omega L \) is diagonal!
POG m-phases Motors:

Types of rotor flux:
1. Trapezoidal
2. Triangular
3. Squared wave
4. Sinusoidal
5. Cosinusoidally connected
6. Sinusoidally connected
7. Polynomial even
8. Polynomial odd
9. Trapezoidal if derived
10. Fourier defined

The stator can also be “star connected”
The POG state space equations (m_s=9, m_r=5):

\[
\begin{bmatrix}
\begin{bmatrix}
\omega L_e & 0 \\
0 & J_m \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\omega I_e \\
\dot{\omega}_m \\
\end{bmatrix}
= 
\begin{bmatrix}
\omega R_e + \omega F_e + \omega \Omega_e & \omega K_e \\
-\omega K_e^T & \frac{1}{b_m} \\
\end{bmatrix}
\begin{bmatrix}
\omega I_e \\
\omega \dot{m} \\
\end{bmatrix}
+ 
\begin{bmatrix}
\omega V_e \\
-\tau_e \\
\end{bmatrix}
\]

The POG scheme in the transformed reference frame:

**Electrical part**

- Transformation
- Energy accumulation
- Energy dissipation
- Energy redistribution

**Mechanical part**

- Energy accumulation
- Energy dissipation
POG Model of Multi-Phase Asynchronous Motors

Two equivalent Simulink implementations of POG multi-phase asynchronous motors:

1) In the transformed reference frame $\Sigma_\omega$:

2) In the original reference frame $\Sigma_t$:
POG model of the Tire-ground Interaction

3D Mechanical dynamics of the wheel

POG Elastic Model

3D Mechanical dynamics of the wheel

POG Elastic Model

UNIMORE 28 Maggio 2018 MORE on Automotive
A parallel Hybrid Automotive System:
- ICE
- PMSM
- Vehicle

Using the POG technique *it is easy to connect the subsystems* because it is based on the use of the “dashed” power sections!
Hybrid Automotive Systems: POG model

The Simulink block scheme:

POG model of PMSM:

PMSM: Permanent Magnet Synchronous Motor

-- Electrical part
-- Mechanical part
-- Energy conversion
Hybrid Automotive Systems: POG model

POG model of the vehicle:
-- From transmission to center of mass
-- Mechanical dynamics
-- From center of mass to road contact

-- Tire-road contact:
  Elasticity
  Skidding
  Slipping
"Start and stop" simulation results obtained with the ICE switched off:

$$\omega_c = 0$$

The small tracking delay is due to the elastic horizontal slipping of the tires on the ground.
A PMSM multiphase electric motor has been used:

\[ m = 5 \]

It can be useful to improve the robustness and safety of the system.

With POG models clearly shows the “power flows” within the system:

- motor
- generator

Hybrid Automotive Systems: simulations
Toyota Hybrid System (THS-II) of the Prius

Internal Combustion Engine

Planetary Gear

Transmission & Vehicle

Batteries are not considered!
Toyota Hybrid System: Simulink scheme

Direct translation of POG schemes (power sections)

Simulation of the Extra Urban Driving Cycle (Normal Operation):

Velocity of the vehicle $\dot{x}_v$

Stator currents zoom: healthy
Simulation of the Extra Urban Driving Cycle

- Fault on MG2 at $t = 145$ s: open phase fault
- Fault tolerant control activation at $t = 160$ s

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Velocity of the vehicle $\dot{v}_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>50</td>
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<tr>
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</tbody>
</table>

- Stator currents zoom: fault at $t = 145$ s
- Stator currents zoom: control at $t = 160$ s

- Planetary gear velocities: sun $\omega_s$, carrier $\omega_c$, and ring $\omega_r$
- Planetary gear torques: sun $\tau_s$, carrier $\tau_c$, and ring $\tau_r$
POG Modeler: the web site of the POG technique

The web site address of the POG Modeler: http://zanasi2009.ing.unimo.it

The Welcome page:

The POG Modeler is free, but you have to be registered

The Login page:

The Home page:
POG Modeler: the Physical Elements

The Internal Physical Elements:

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>( eC )</td>
<td>Capacitor</td>
<td>( v \rightarrow i )</td>
<td>( M \rightarrow \omega )</td>
<td>( C \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( eL )</td>
<td>Inductor</td>
<td>( v \rightarrow i )</td>
<td>( E \rightarrow \omega )</td>
<td>( L \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( eR )</td>
<td>Resistor</td>
<td>( v \rightarrow i )</td>
<td>( d \rightarrow \omega )</td>
<td>( R \rightarrow \omega_i )</td>
</tr>
</tbody>
</table>

The Internal and the External Physical Elements “PE” are identified by using a two digit string “\( xX \)”: 

The External Physical Elements:

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( mM )</td>
<td>Mass</td>
<td>( F \rightarrow \omega )</td>
<td>( J \rightarrow \omega_i )</td>
<td>( M \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( mE )</td>
<td>Spring</td>
<td>( F \rightarrow \omega )</td>
<td>( K \rightarrow \omega_i )</td>
<td>( E \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( mD )</td>
<td>Friction</td>
<td>( F \rightarrow \omega )</td>
<td>( B \rightarrow \omega_i )</td>
<td>( d \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( mF )</td>
<td>Force Gen.</td>
<td>( F \rightarrow \omega )</td>
<td>( \tau \rightarrow \omega_i )</td>
<td>( F \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( mT )</td>
<td>Torque Gen.</td>
<td>( \tau \rightarrow \omega )</td>
<td>( Q \rightarrow \omega_i )</td>
<td>( T \rightarrow \omega_i )</td>
</tr>
<tr>
<td>( iQ )</td>
<td>Flow Rate Gen.</td>
<td>( Q \rightarrow \omega )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Across-generators \( \mathcal{G}_a \):
- Voltage Gen.
- Velocity Gen.
- Ang. Velocity Gen.
- Pressure Gen.

Through-generators \( \mathcal{G}_t \):
- Current Gen.
- Force Gen.
- Torque Gen.
- Flow Rate Gen.
POG Modeler: a simple electric circuit

The Physical System are defined using “ascii command-lines”:

- **eV**, a, b, An, -90, En, Vin
- **eR**, a, c, Kn, Ra
- **- -**, b, d
- **eC**, c, d, En, Vb, Kn, Cb
- **eR**, c, e, Kn, Rc
- **- -**, d, f
- **eL**, e, f, Fn, Id, Kn, Ld
- **eR**, e, f, Sh, 0.6, Kn, RL

Each line defines one Physical Element:

1) The string “eV, a, b, En, Vin” defines the Voltage Generator Vin between points a and b;
2) The string “eR, a, c, Kn, Ra” defines the Resistance Ra between points a and c;
3) The string “- -, b, d” defines the line connecting point b to point d;
4) The string “eC, c, d, En, Vb, Kn, Cb” defines the Capacitor Cb between points c and d;
5) The string “eR, c, e, Kn, Rc” defines the Resistance Rc between points c and e;
6) The string “- -, d, f” defines the line connecting point d to point f;
7) …
POG Modeler: POG and Simulink schemes

Command “As, Yes” provides the state space equations in symbolic form:

% State space equations:
% \( \dot{X} = AM \cdot X + BM \cdot U \)
% \( Y = CM \cdot X + DM \cdot U \)
LM = ... % Energy matrix LM:
[ Cb, 0]
[ 0, Ld]
AM = ... % Power matrix AM:
[ -1/(RL + Rc) - 1/Ra, -RL/(RL + Rc)]
[ RL/(RL + Rc), -(RL*Rc)/(RL + Rc)]
BM = ... % Input matrix BM:
1/Ra
0
CM = ... % Output matrix CM:
[ -1/Ra, 0]
DM = ... % Input-output matrix DM:
1/Ra
X = ... % State vector X:
Vb
Id
U = ... % Input vector U:
Vin
Y = ... % Output vector Y:
I_1

Command “POG, Yes” provides the POG block scheme:

Command “SLX, Yes” provides the Simulink block scheme:
POG Modeler: an Hydraulic System

The hydraulic circuit:

The corresponding POG block scheme is:

The command lines:

**, Gr, Si, Sn, Si, As, No,
** , EQN, No, POG, No
iP, 1, a, En, P a, An, -90
iR, 1, 2, Kn, R 1, En, P 1, Ln, 0.6
iL, 2, 3, Kn, L 1, Fn, Q 1
iC, 3, b, Kn, C 2, En, P 2, An, -90
- - , a, b
- - , 3, 4, Ln, 0.4
iR, 3, b, Kn, R 2, Fn, -Q 2, Sh, 0.4
iL, 4, 5, Kn, L 3, Fn, -Q 3, Ln, 1.0
- - , b, c, Ln, 1.4
iC, 5, c, Kn, C 4, En, P 4, An, -90
iR, 5, c, Kn, R 4, Fn, -Q 4, Sh, 0.4
iQ, c, 5, Fn, Q b, Sh, -0.9
POG Modeler: the KERS System

rT, 1, a, An, -90, Ln, 1.2
rJ, 1, a, Sh, 0.5, Kn, J_1, En, w_1, Kn0, 13, Qn0, 2
\rightarrow, 1, 2
\rightarrow, a, b
rB, 2, b, Kn, b_1
CB, [2;3],[b;c], Kn, E2=r_1*E1, Sh, [0.6;0.2]
mK, 3, 4, Ln, 0.8, Kn, K_1.2
\rightarrow, c, d, Ln, 0.8
CB, [4;5],[d;e], Kn, E1=r_2*E2, Sh, [0.2;0.6]
rJ, 5, e, Kn, J_2, En, w_2
\rightarrow, 5, 6, Ln, 0.5
\rightarrow, e, f, Ln, 0.5
rB, 6, f, Kn, b_2
CB, [6;7],[f;g], Kn, E2=r_2_t_h*E1, Sh, [0.6;0.2]
mK, 7, 8, Ln, 0.8, Kn, K_2.b
\rightarrow, g, h, Ln, 0.8
CB, [8;9],[h;i], Kn, F2=r_b*F1, Sh, [0.2;0.6]
rJ, 9, i, Kn, J_b, En, w_b

...
POG Modeler: a complex electric system

\[ \text{eV, 1, a, En, V_a, Fn, I_a, An, -90, Ln, 3'} \]
\[ \text{eR, 1, 2, Kn, R_1, Ln, 1, Out, Flow' \]
\[ \text{eC, 2, 3, Kn, C_1, En, V_1, An, -90, Ln, 1.5'} \]
\[ \text{... a, b'} \]
\[ \text{eR, 3, b, Kn, R_2, Ln, 1.5, Out, Flow' \]
\[ \text{eL, 3, b, Kn, L_1, Fn, I_1, Sh, 0.6' \]
\[ \text{eR, 3, 4, Kn, R_3, Ln, 1.6, Tr, 0.3' \]
\[ \text{... b, c, Ln, 1.6' \]
\[ \text{eC, 4, c, Kn, C_2, En, V_2' \]
\[ \text{eL, 2, 5, Kn, L_2, Fn, I_2,Ln, 1.2' \]
\[ \text{eV, 2, 5, En.-V_b, Fn, I_b, Sh, 0.5' \]
\[ \text{eR, 5, 6, Kn, R_4, Ln, 1.2, Out, Flow' \]
\[ \text{eC, 6, 7, Kn, C_3, En, V_3, An, -90, Ln, 1.0' \]
\[ \text{eR, 6, 7, Kn, R_5, Sh, 0.6' \]
\[ \text{eL, 7, 8, Kn, L_3, Fn, I_3, An, -90' \]
\[ \text{eR, 7, 8, Kn, R_6, Sh, 0.6, Out, Flow' \]
\[ \text{... c, d, Ln, 0.8' \]
\[ \text{eR, 8, d, Kn, R_7' \]
\[ \text{... 7, 9, Ln, 1.2' \]
\[ \text{... d, e, Ln, 1.2' \]
\[ \text{eC, 9, e, Kn, C_4, En, V_4' \]
\[ \text{eI, 9, e, Fn, I_c, En, V_c, Sh, 0.8'} \]
POG Modeler: Basic Features

Today Features:
1) Energetic domains:
   a) Electrical;
   b) Mechanical Translational;
   c) Mechanical Rotational;
   d) Hydraulic.
2) Provides all types of Transformers and Gyrators
3) Only for Linear Time-Invariant Systems
4) Only for One-dimensional Variables
5) Command line ascii interface

Future work:
1) The Thermic Energetic Domain;
2) Multi-dimensional Variables
3) Graphical Interface
4) Also for Nonlinear Time-Variant Systems
5) Provide other graphical outputs: Bond Grahs (BG) and Energetic Macroscopic Representation (EMR)
Conclusions

• The Power-Oriented Graphs (POG) technique is a very powerful and simple tool for Modelling Physical Systems.

• In particular it is very useful for modelling Automotive Systems and Subsystems.

• The POG technique is also good for beginners.

• If you like the POG modelling technique, use on the web the POG modeler:

  http://zanasi2009.ing.unimo.it
Thank You!

MORE on Automotive - 28 Maggio 2018
Prof. Roberto Zanasi